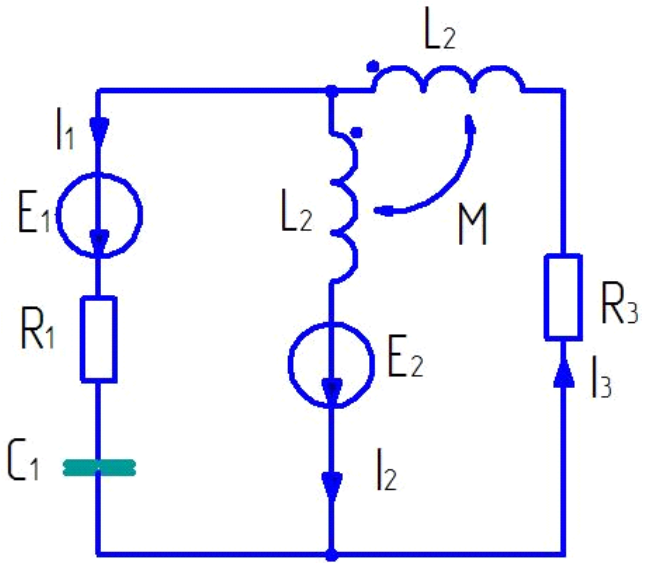
РГР 2
Вариант 1
Задача 3



$E_1 = \frac{39}{\sqrt{2}} \cdot e^{j \cdot 20deg} = 25.914 + 9.432j$ В

$E_2 = \frac{85}{\sqrt{2}} = 60.104$ В

$R_1 = 50$ Ом

$C_1 = 20 \cdot 10^{-6}$ Ф

$L_2 = 0.064$ Гн

$L_3 = 0.1$ Гн

$R_3 = 27$ Ом

$f = 500$ Гц

$M = 0.040$ Гн

Решение

$X_{L2} = j \cdot 2 \cdot \pi \cdot f \cdot L_2 = j \cdot 2 \cdot \pi \cdot 500 \cdot 0.064 = 201.062j$ Ом

$X_{L3} = j \cdot 2 \cdot \pi \cdot f \cdot L_3 = j \cdot 2 \cdot \pi \cdot 500 \cdot 0.1 = 314.159j$ Ом

$X_M = j \cdot 2 \cdot \pi \cdot f \cdot M = j \cdot 2 \cdot \pi \cdot 500 \cdot 0.04 = 125.664j$ Ом

$X_{C1} = \frac{1}{j \cdot 2 \cdot \pi f \cdot C_1} = \frac{1}{j \cdot 2 \cdot \pi \cdot 500 \cdot 20 \cdot 10^{-6}} = -15.915j$ Ом

Составим систему уравнений по законам Кирхгоффа

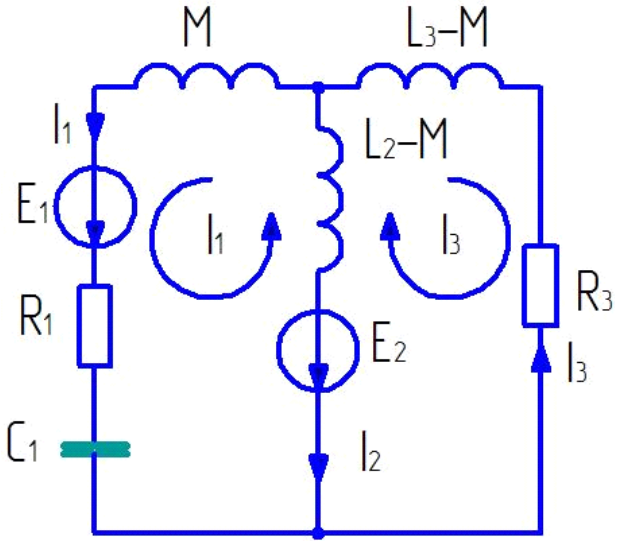
$I_1 + I_2 - I_3 = 0$

$I_1 \cdot (X_{C1} + R_1) + I_3 \cdot (X_{L3} + R_3) - I_2 \cdot X_M = E_1$

$I_2 \cdot X_{L2} + I_3 \cdot X_M + I_3 \cdot (X_{L3} + R_3) - I_2 \cdot X_M = E_2$

Развяжем индуктивную связь

По полученной схеме составим систему уравнений по методу контурных токов



$$I_1\cdot\left(X_{L2} - X_M + X_M + R_1 + X_{C1}\right) - I_3\cdot\left(X_{L2} - X_M\right) = E_1 - E_2$$
$$-I_1\cdot\left(X_{L2} - X_M\right) + I_2\cdot\left(X_{L2} - X_M + X_{L3} - X_M + R_3\right) = -E_2$$

$$I_1\cdot\left(X_{L2} + R_1 + X_{C1}\right) - I_3\cdot\left(X_{L2} - X_M\right) = E_1 - E_2$$
$$-I_1\cdot\left(X_{L2} - X_M\right) + I_2\cdot\left(X_{L2} - 2X_M + X_{L3} + R_3\right) = -E_2$$

Рассчитаем токи при помощи матриц

$$M_1 = \begin{bmatrix} X_{L2} + R_1 + X_{C1} & -\left(X_{L2} - X_M\right) \\ -\left(X_{L2} - X_M\right) & X_{L2} - 2X_M + X_{L3} + R_3 \end{bmatrix}$$

$$M_1 = \begin{bmatrix} 50 + 201.062j + -15.915j & -(201.062j - 125.664j) \\ -(201.062j - 125.664j) & 201.062j - 2\cdot 125.664j + 314.159j + 27 \end{bmatrix}$$

$$M_1 = \begin{pmatrix} 50 + 185.146j & -75.398j \\ -75.398j & 27 + 263.894j \end{pmatrix}$$

$$M_2 = \begin{pmatrix} E_1 - E_2 \\ -E_2 \end{pmatrix} = \begin{pmatrix} -34.19 + 9.432j \\ -60.104 \end{pmatrix}$$

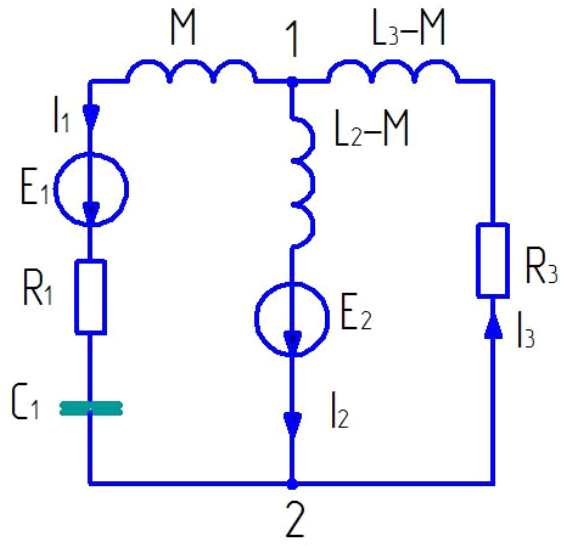
$$M_3 = M_1^{-1}\cdot M_2 = \begin{pmatrix} -0.048 + 0.297j \\ -0.045 + 0.308j \end{pmatrix}$$

$I_1 = M_{3_1} = -0.048 + 0.297j$ A $\quad |I_1| = 0.301$ A $\quad \angle(I_1) = 99.12$

$I_3 = M_{3_2} = -0.045 + 0.308j$ A $\quad |I_3| = 0.311$ A $\quad \angle(I_3) = 98.338$

$I_2 = I_1 + I_3 = -0.093 + 0.605j$ A $\quad |I_2| = 0.612$ A $\quad \angle(I_2) = 98.722$

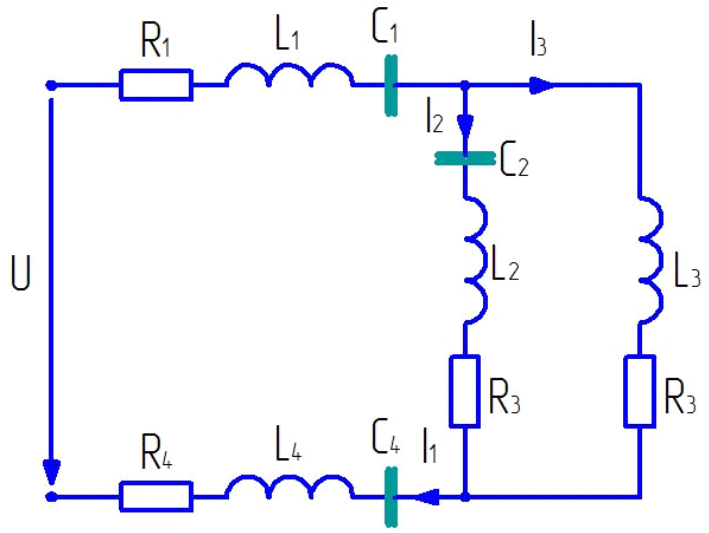
Составим систему уравнений по методу узловых потенциалов



$$\varphi_1 = 0 \quad \text{В}$$

$$\varphi_2 = \frac{\dfrac{E_1}{X_M + R_1 + X_{C1}} + \dfrac{E_2}{X_{L2} - X_M}}{\dfrac{1}{X_M + R_1 + X_{C1}} + \dfrac{1}{X_{L2} - X_M} + \dfrac{1}{X_{L3} - X_M + R_3}}$$

Задача 1



$R_1 = 1$ Ом

$X_{L1} = 1$ Ом

$X_{C1} = 4$ Ом

$R_2 = 1$ Ом

$X_{L2} = 1$ Ом

$X_{C2} = 2$ Ом

$R_3 = 1$ Ом

$X_{L3} = 1$ Ом

$X_{C4} = 2$ Ом

$R_4 = 1$ Ом

$X_{L4} = 2$ Ом

$u(t) = 10 \cdot \sin(314t - 30\text{deg})$ В

Решение

Рассчитаем действующие значения токов и напряжений ветвей методом проводимостей

$$Z_3 = \sqrt{R_3^2 + X_{L3}^2} = \sqrt{1^2 + 1^2} = 1.414 \quad \text{Ом}$$

$$g_3 = \frac{R_3}{Z_3^2} = \frac{1}{1.414^2} = 0.5 \quad \text{См}$$

$$b_3 = \frac{X_{L3}}{Z_3^2} = \frac{1}{1.414^2} = 0.5 \quad \text{См}$$

$$Z_2 = \sqrt{R_2^2 + (X_{L2} - X_{C2})^2} = \sqrt{1^2 + (1 - 2)^2} = 1.414 \quad \text{Ом}$$

$$g_2 = \frac{R_2}{Z_2^2} = \frac{1}{1.414^2} = 0.5 \quad \text{См}$$

$$b_2 = \frac{X_{L2} - X_{C2}}{Z_2^2} = \frac{1 - 2}{1.414^2} = -0.5 \quad \text{См}$$

$$y_{23} = \sqrt{(g_2 + g_3)^2 + (b_2 + b_3)^2} = \sqrt{(0.5 + 0.5)^2 + (-0.5 + 0.5)^2} = 1 \quad \text{См}$$

$$Z_{23} = \frac{1}{y_{23}} = \frac{1}{1} = 1 \quad \text{Ом}$$

$$R_{23} = \frac{g_2 + g_3}{y_{23}^2} = \frac{0.5 + 0.5}{1^2} = 1 \quad \text{Ом}$$

$$X_{23} = \frac{b_2 + b_3}{y_{23}^2} = \frac{-0.5 + 0.5}{1^2} = 0 \quad \text{Ом}$$

$$Z_1 = \sqrt{R_1^2 + \left(X_{L1} - X_{C1}\right)^2} = \sqrt{1^2 + (1 - 4)^2} = 3.162 \quad \text{Ом}$$

$$Z_4 = \sqrt{R_4^2 + \left(X_{L4} - X_{C4}\right)^2} = \sqrt{1^2 + (2 - 2)^2} = 1 \quad \text{Ом}$$

$$Z = \sqrt{\left(R_1 + R_{23} + R_4\right)^2 + \left(X_{L1} - X_{C1} + X_{L4} - X_{C4} + X_{23}\right)^2} = \sqrt{(1 + 1 + 1)^2 + (1 - 4 + 2 - 2 + 0)^2} = 4.243 \text{ Ом}$$

$$U = \frac{10}{\sqrt{2}} = 7.071 \quad \text{В}$$

$$I_1 = \frac{U}{Z} = \frac{7.071}{4.243} = 1.667 \quad \text{А}$$

$$U_1 = I_1 \cdot Z_1 = 1.667 \cdot 3.162 = 5.27 \quad \text{В}$$

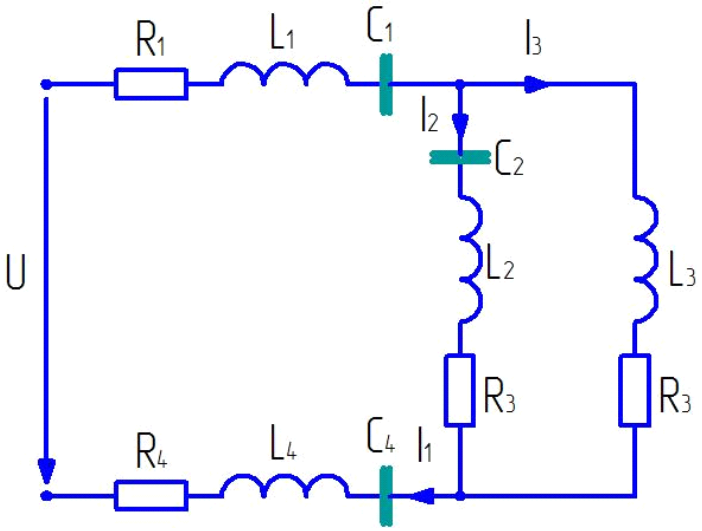
$$U_{23} = I_1 \cdot Z_{23} = 1.667 \cdot 1 = 1.667 \quad \text{В}$$

$$U_4 = I_1 \cdot Z_4 = 1.667 = 1.667 \quad \text{В}$$

$$I_2 = \frac{U_{23}}{Z_2} = \frac{1.667}{1.414} = 1.179 \quad \text{А}$$

$$I_3 = \frac{U_{23}}{Z_3} = \frac{1.667}{1.414} = 1.179 \quad \text{А}$$

Рассчитаем токи и напряжения символическим методом



$$U = \frac{10}{\sqrt{2}} \cdot e^{-j \cdot 30deg} = 6.124 - 3.536j \quad B$$

$$Z_1 = R_1 + j \cdot X_{L1} - j \cdot X_{C1} = 1 + j - j \cdot 4 = 1 - 3j \quad Ом$$

$$Z_2 = R_2 + j \cdot X_{L2} - j \cdot X_{C2} = 1 + j - j \cdot 2 = 1 - j \quad Ом$$

$$Z_3 = R_3 + j \cdot X_{L3} = 1 + j = 1 + j \quad Ом$$

$$Z_4 = R_4 + j \cdot X_{L4} - j \cdot X_{C4} = 1 + j \cdot 2 - j \cdot 2 = 1 \quad Ом$$

$$I_1 = \frac{U}{Z_1 + \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} + Z_4} = \frac{6.124 - 3.536j}{1 - 3j + \frac{(1 + j) \cdot (1 - j)}{1 - j + 1 + j} + 1} = 1.61 + 0.431j \quad A$$

$$|I_1| = 1.667 \quad A \quad \angle(I_1) = 15$$

$$U_1 = I_1 \cdot Z_1 = (1 - 3j) \cdot (1.61 + 0.431j) = 2.904 - 4.398j \quad B \quad |U_1| = 5.27 \quad B \quad \angle(U_1) = -56.565$$

$$U_4 = I_1 \cdot Z_4 = (1.61 + 0.431j) = 1.61 + 0.431j \quad B \quad |U_4| = 1.667 \quad B \quad \angle(U_4) = 15$$

$$U_{23} = I_1 \cdot \frac{Z_2 \cdot Z_3}{Z_2 + Z_3} = (1.61 + 0.431j) \cdot \frac{(1 + j) \cdot (1 - j)}{1 - j + 1 + j} = 1.61 + 0.431j \quad B \quad |U_{23}| = 1.667 \quad B \quad \angle(U_{23}) = 15$$

$$I_2 = \frac{U_{23}}{Z_2} = \frac{1.61 + 0.431j}{1 - j} = 0.589 + 1.021j \quad A \quad |I_2| = 1.179 \quad A \quad \angle(I_2) = 60$$

$$I_3 = \frac{U_{23}}{Z_3} = \frac{1.61 + 0.431j}{1 + j} = 1.021 - 0.589j \quad A \quad |I_3| = 1.179 \quad A \quad \angle(I_3) = -30$$

Построим топографическую диаграмму напряжений совмещенную с векторной диаграммой токов